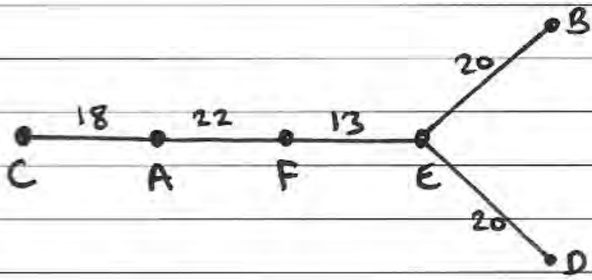
D2 June 2010

1. (a) and (b)

	A	B	C	D	E	F
A	-	36	18	28	24	22
B	36	-	54	22	20	27
C	18	54	-	42	27	24
D	28	22	42	-	20	30
E	24	20	27	20	-	13
F	22	27	24	30	13	-

Prim's AC(18)
AF(22)
FE(13)
EB(20)
ED(20)
93



b) Initial upper bound = 93×2 = 186

(c)

	A	B	C	D	E	F
~~A~~	~~-~~	~~36~~	~~18~~	~~28~~	~~24~~	~~22~~
B	36	-	54	22	20	27
~~C~~	18	~~54~~	~~-~~	~~42~~	~~27~~	~~24~~
D	28	22	42	-	20	30
~~E~~	~~24~~	~~20~~	~~27~~	~~20~~	~~-~~	13
~~F~~	~~22~~	~~27~~	24	~~30~~	~~13~~	~~-~~

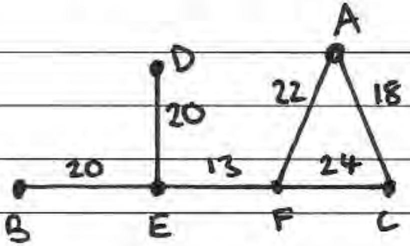
$A_{18}C_{24}F_{13}E_{20}B_{22}D_{28}A \to 125$

$A_{18}C_{24}F_{13}E_{20}D_{22}B_{36}A \to 133$

(d) Best upper bound: 125 ACFEBDA

(e) Smaller, closer to optima

	A	B	C	D	E	F
A	-	36	18	28	24	22
B	36	-	54	22	20	27
C	18	54	-	42	27	24
D	28	22	42	-	20	30
E	24	20	27	20	-	13
F	22	27	24	30	13	-



Prim's BE(20)
EF(13)
ED(20)
CF(24)

RMST 77

+ AC(18) + Af(22) ⇒ lower bound = 117

117 < optimal route ≤ 125

(Total 11 marks)

2. *You may not need to use all of these tables*

(a)

	1	2	3	4
H	18	24	22	17
J	20	25	19	–
L	25	24	27	22
S	19	26	23	14

Maximise
∴ subtract every element from 27

	1	2	3	4
H	9	3	5	10
J	7	2	8	20
L	2	3	0	5
S	8	1	4	13

J4 = 2x biggest element so that it is not selected.

Reduce Rows

	1	2	3	4	
H	6	0	2	7	−3
J	5	0	6	18	−2
L	2	3	0	5	x
S	7	0	3	12	−1

Reduce Columns

	1	2	3	4
H	4	0	2	2
J	3	0	6	13
L	0	3	0	0
S	5	0	3	7
	−2	x	x	−5

Smallest uncovered = 2

2 lines ⇒ not optimal

	1	2	3	4
H	2	0	0	0
J	1	0	4	11
L	0	5	0	0
S	3	0	1	5

3 lines ⇒ not

Smallest uncovered

	1	2	3	4
H	2	1	0	0
J	0	0	3	10
L	0	6	0	0
S	2	0	0	4

4 lines required

∴ optimal

	1	2	3	4
H			0	0
J	0	0		
L	0		0	0
S		0	0	

(b)

Worker	Task	
Harry	3	(22)
Jess	1	(20)
Louis	4	(22)
Saul	2	(26)

£ 90

3. (a)

	A	B	C	D	Supply
X	18	31	4		53
Y			18	29	47
Demand	18	31	22	29	

Initi[al] Soluti[on]

(b) *You may not need to use all of these tables*

		A	B	C	D
		28	20	19	22
0	X	28	20	19	16
-5	Y	15	12	14	17

Shadow Costs

	A	B	C	D
X	X	X	X	-6
Y	-8	-3	X	X

Improvement Indices

	A	B	C	D
X	18 $-\theta$	31	4 $+\theta$	
Y	$+\theta$		18 $-\theta$	29

entering cell = YA

$\theta = 18$

exiting cell = XA

put 0 in YC so it is not degenerate

	A	B	C	D
X		31	22	
Y	18		0	29

New Solution

		A	B	C	D
		20	20	19	22
0	X	28	20	19	16
-5	Y	15	12	14	17

Shadow Costs

	A	B	C	D
X	8	x	x	−6
Y	x	−3	x	x

Improvement Indices

	A	B	C	D
X		31	$22^{-\theta}$	$+\theta$
Y	18		$0^{+\theta}$	$29^{-\theta}$

entering cell = XD

$\theta = 22$

exiting cell = XC

	A	B	C	D
X		31		22
Y	18		22	7

New Solution

		14	20	13	16
		A	B	C	D
0	X	28	20	19	16
1	Y	15	12	14	17

Shadow Costs

(c)

Improvement Indices

$XA = 14$; $XC = 6$; $YB = -9$

$\therefore$ Not optimal as there is still a negative improvement index.

4. (a) *You may not need to use all these rows*

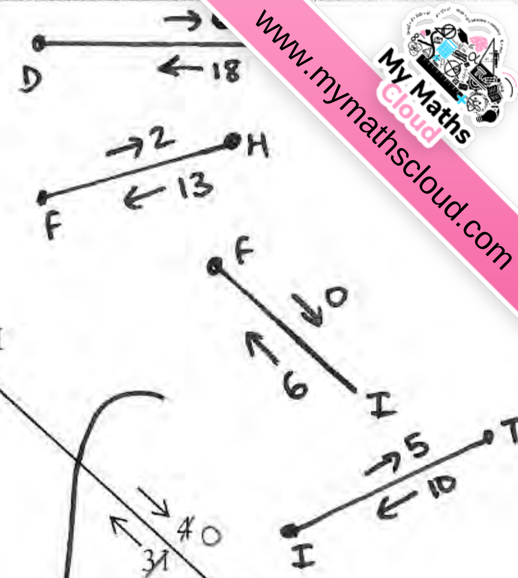
minimax

Stage	State	Action	Destination	Value
1	G	GT	T	17 *
	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	max(22, 17) = 22 *
		DH	H	max(31, 21) = 31
	E	EH	H	max(34, 21) = 34 *
		EI	I	max(39, 29) = 39
	F	FI	I	max(52, 29) = 52 *
3	A	AD	D	max(41, 22) = 41
		AE	E	max(38, 34) = 38 *
	B	BE	E	max(44, 34) = 44 *
	C	CE	E	max(36, 34) = 36 *
		CF	F	max(35, 52) = 52
4	S	SA	A	max(37, 38) = 38 *
		SB	B	max(39, 44) = 44
		SC	C	max(41, 36) = 41
				SAEHT £38000
				S A E H T 130
				37 38 34 21
				average = 32.5
				£32500

5. (a) Value of initial flow: $8 + 3 + 30 = 41$

(b) Capacity of C_1: $8 + 15 + 10 + 11 + 25 = 69$

Capacity of C_2: $24 + 8 + 10 + 18 + 4 = 64$



(c)

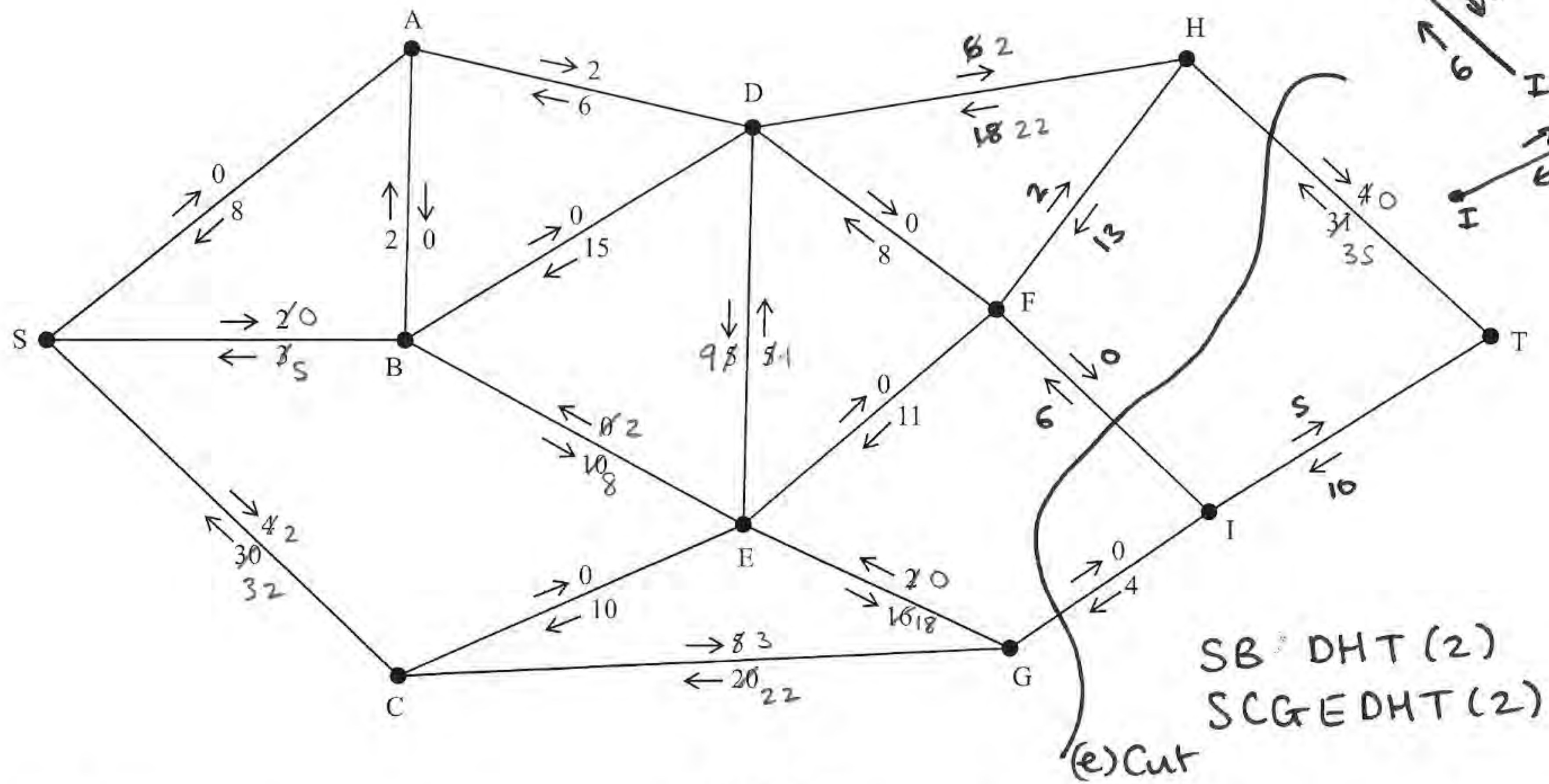


Diagram 1

Cut through HT, FI, GI Now only passes through saturated arcs.

$35 + 6 + 4 = 45$ ∴ min cut $= 45$ by min cut - max flow theorem max flow $= 45$

∴ flow is maximal

Leave blank

6. (a) $P - x - 2y - 6z = 0$

	b.v.	x	y	z	r	s	t	Value
	r	0	1	2	1	0	0	24
*	s	2	1	4	0	1	0	28
	t	–1	$\frac{1}{2}$	3	0	0	1	22
	P	–1	–2	–6	0	0	0	0

$\theta = 24 \div 2 = 12$

$\theta = 28 \div 4 = 7$ *

$\theta = 22 \div 3 = 7\frac{1}{3}$

You may not need to use all of these tableaux

(b) Increase z

	b.v.	x	y	z	r	s	t	Value	Row Ops
		0	1	2	1	0	0	24	x
*		1/2	1/4	1	0	1/4	0	7	R2 ÷ 4
		-1	1/2	3	0	0	1	22	x
	P	-1	-2	-6	0	0	0	0	x

	b.v.	x	y	z	r	s	t	Value	Row Ops	
*	r	-1	1/2	0	1	-1/2	0	10	- 2R2	$\theta = 10 \div \frac{1}{2} = 20$*
	z	1/2	1/4	1	0	1/4	0	7	x	$\theta = 7 \div \frac{1}{4} = 28$
	t	-5/2	-1/4	0	0	-3/4	1	1	-3R2	θ -ve.
	P	2	-1/2	0	0	3/2	0	42	+6R2	

Question 6 continued

Increasing y

b.v.	x	y	z	r	s	t	Value	Row Ops
	-2	1	0	2	-1	0	20	$R1 \div \frac{1}{2}$ (x2)
	1/2	1/4	1	0	1/4	0	7	X
	-5/2	-1/4	0	0	-3/4	1	1	X
P	2	-1/2	0	0	3/2	0	42	X

b.v.	x	y	z	r	s	t	Value	Row Ops
y	-2	1	0	2	-1	0	20	X
z	0	0	1	$-\frac{1}{2}$	1/2	0	2	$-\frac{1}{4}R1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$+\frac{1}{4}R1$
P	1	0	0	1	1	0	52	$+\frac{1}{2}R1$

$x=0$
$y=20$
$z=2$
$r=0$
$s=0$
$t=6$
$P=52$

7.

	B plays 1	B plays 2	B plays 3
A plays 1	–4	5	1
A plays 2	3	–1	–2
A plays 3	–3	0	2

v = value of original game to A

+5 to every element so that each is > 0

	B1	B2	B3
A1	1	10	6
A2	8	4	3
A3	2	5	7

now let $V = v + 5$ be the value of the new game to A.

A plays 1 prob = p_1
A plays 2 prob = p_2 $\quad p_1, p_2, p_3 \geq 0$
A plays 3 prob = p_3

If B plays 1 $\quad V = p_1 + 8p_2 + 2p_3$
B plays 2 $\quad V = 10p_1 + 4p_2 + 5p_3$
B plays 3 $\quad V = 6p_1 + 3p_2 + 7p_3$

$\therefore$ objective is to maximise $P = V$ such that $P - V = 0$

Subject to

$V \leq p_1 + 8p_2 + 2p_3$
$V \leq 10p_1 + 4p_2 + 5p_3$
$V \leq 6p_1 + 3p_2 + 7p_3$
$p_1 + p_2 + p_3 \leq 1 \qquad p_1, p_2, p_3 \geq 0$